السلام عليكم

دة حل مسألة الكويز الاخير (الرابع) بثلاث طرق مختلفة
كل طرقة بعدد neuron مختلفة وطبعا عدد weights
مختلف وافضل طريقة اللى لها عدد neuron اقل
وبالتالى يكون عدد weights اقل ايضا

ممكن الدكتور يطلب حل بعينة اى يحدد عدد النيورون في المسألة

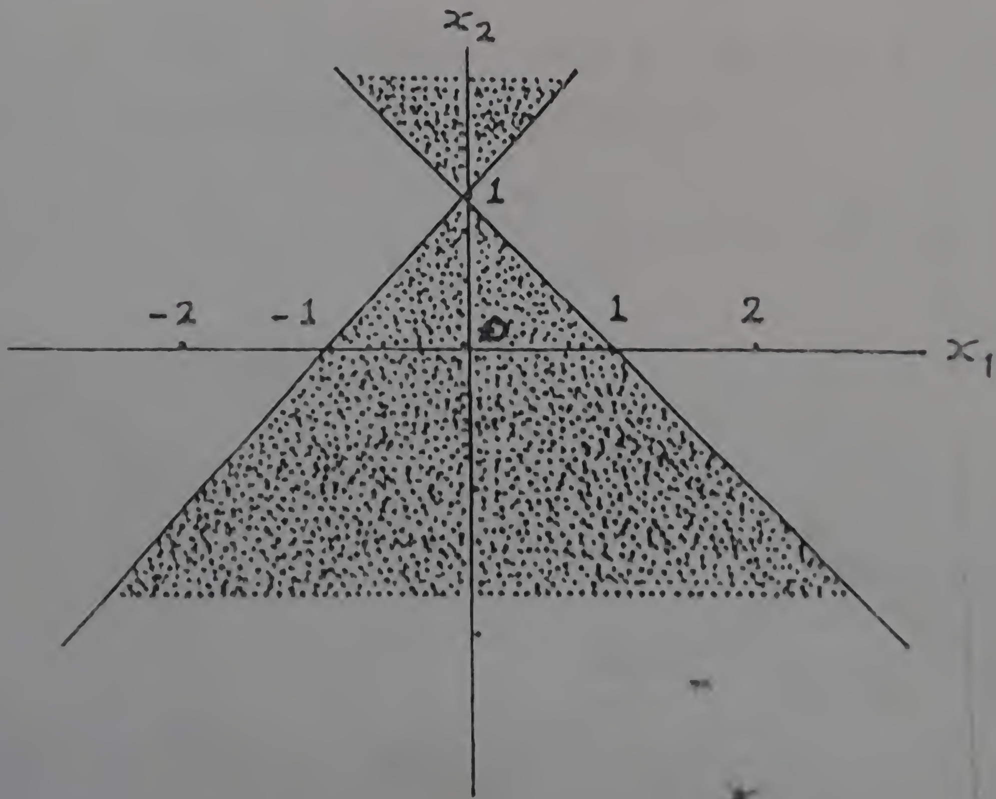
Tanta University
Faculty of Engineering
Computers and Control Department

Fourth Year Students
Academic Year 2016/2017
First Semester

NEURAL NETWORKS *Quiz No.4*

Solve the following problem. Time allowed: 30 minutes.

Design a neural network, with two inputs x_1 and x_2 and a single output s, that behaves as a two-class data classifier. On the $x_1 - x_2$ plane, shown below, all input patterns (x_1, x_2) inside the two shaded areas are identified by an output value $s = 1$, whereas all input patterns outside these areas are identified by $s = 0$. How will your network classify the input patterns (0,2), (0,-2), (2,1), and (-3,0). Can the network properly classify the input pattern (1,2)? Why?

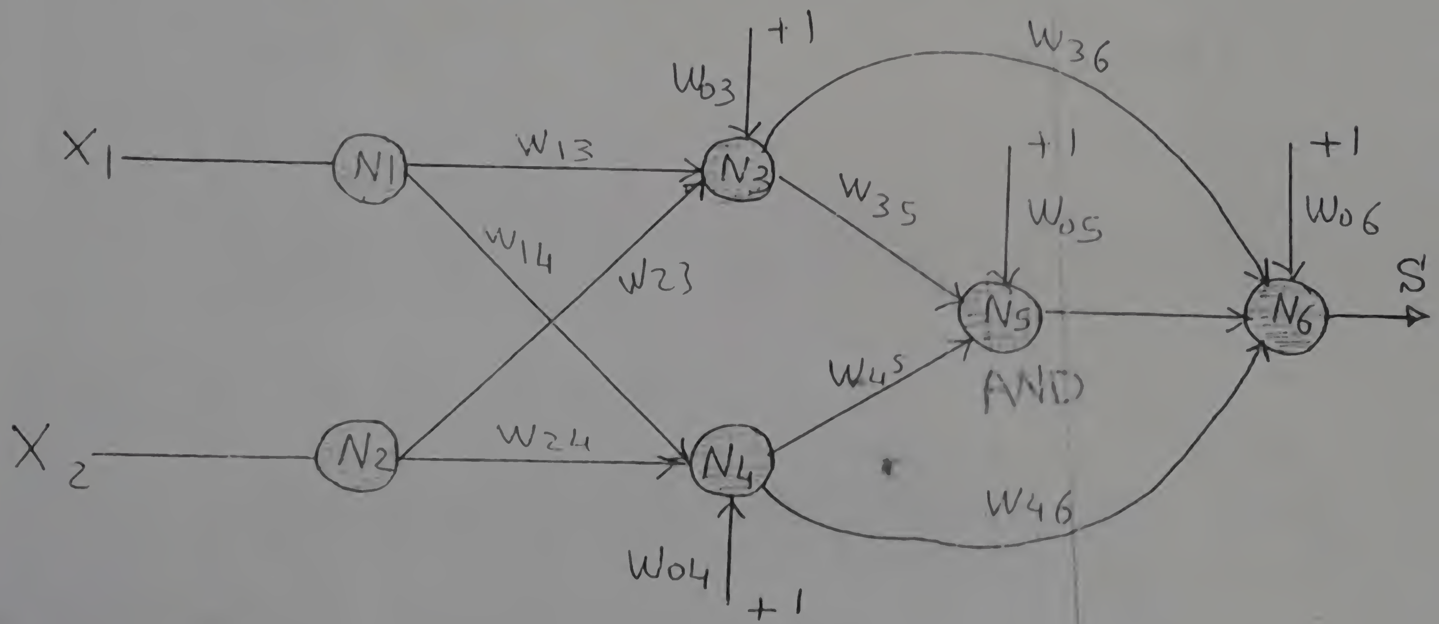


BEST WISHES

Prof. Dr. Mahmoud M. Fahmy
December 14, 2016

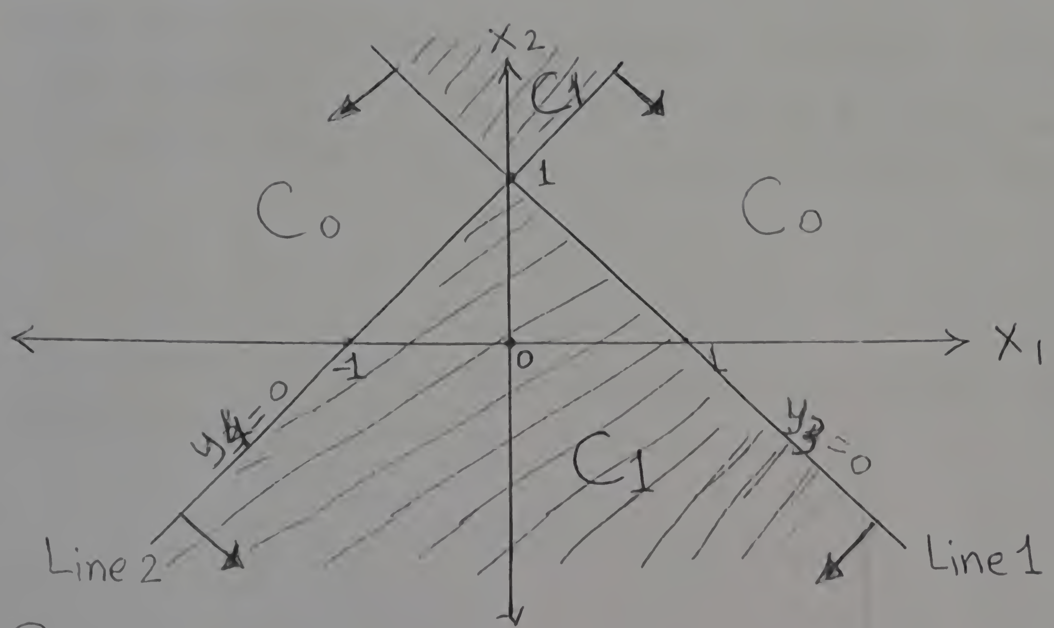
Solution 1

The neural network consists of four layers: an input layer with two neurons N1 and N2; two hidden layers, the first with two neurons N3 and N4 and the second with one neuron N5; and an output layer with a single neuron N7.
All neurons in the hidden and output layers employ binary threshold activation signals.



The orientation of the two separation lines are taken as indicated.

- Neurons N5 and N6 perform logic operation XNOR
- Neuron N5 performs logic operation AND



Line 1 $(0,1), (1,0)$

$$\frac{x_1-0}{x_2-1} = \frac{1-0}{0-1} \Rightarrow \boxed{-x_1-x_2+1=0}$$

Line 2 $(0,1), (-1,0)$

$$\frac{x_1-0}{x_2-1} = \frac{0-(-1)}{1-0} \Rightarrow \boxed{x_1-x_2+1=0}$$

* separation Line(1) $\longrightarrow y_3=0$

$$y_3 = w_{13}x_1 + w_{23}x_2 + w_{03}$$
$$= -x_1 - x_2 + 1$$

$\boxed{w_{13}=-1}$ $\boxed{w_{23}=-1}$ $\boxed{w_{03}=1}$

* separation Line(2) $\longrightarrow y_4=0$

$$y_4 = w_{14}x_1 + w_{24}x_2 + w_{04}$$
$$= x_1 - x_2 + 1$$

$\boxed{w_{14}=1}$ $\boxed{w_{24}=-1}$ $\boxed{w_{04}=1}$

S1 (2)

N5 and N6 performs a Logic XNOR operation on The Signals $f(y_3)$ and $f(y_4)$ produced by neurons N3 and N4, respectively; That is

$$\boxed{S = f(y_3) \odot f(y_4)}$$

* We make neuron N5 Perform an AND operation; That is,

$$\boxed{f(y_5) = f(y_3) \cdot f(y_4)}$$

activation of N5,

$$y_5 = W_{35} f(y_3) + W_{45} f(y_4) + W_{05}$$

$f(y_3)$	$f(y_4)$	$f(y_3) f(y_4)$	S
0	0	0	1
0	1	0	0
1	0	0	0
1	1	1	1

For $f(y_3) = 0$ and $f(y_4) = 0$,
$y_5 = W_{05} < 0$
For $f(y_3) = 0$ and $f(y_4) = 1$;
$y_5 = W_{45} + W_{05} < 0$
For $f(y_3) = 1$ and $f(y_4) = 0$,
$y_5 = W_{35} + W_{05} < 0$
For $f(y_3) = 1$ and $f(y_4) = 1$,
$y_5 = W_{35} + W_{45} + W_{05} > 0$
We choose:

$\boxed{W_{35} = 1}$ $\boxed{W_{45} = 1}$ $\boxed{W_{05} = -1.5}$

and we get $y_5 = f(y_3) + f(y_4) - 1.5$

Activation of N6,

$$y_6 = w_{36} f(y_3) + w_{46} f(y_4) + w_{56} f(y_5) + w_{06}$$

For $f(y_3) = 0$, $f(y_4) = 0$ and $f(y_5) = 0$

$$y_6 = w_{06} > 0$$

For $f(y_3) = 0$, $f(y_4) = 1$ and $f(y_5) = 0$

$$y_6 = w_{46} + w_{06} < 0$$

For $f(y_3) = 1$, $f(y_4) = 0$ and $f(y_5) = 0$

$$y_6 = w_{36} + w_{06} < 0$$

for $f(y_3) = 1$, $f(y_4) = 1$ and $f(y_5) = 1$

$$y_6 = w_{36} + w_{46} + w_{56} + w_{06} > 0$$

We choose:

$\boxed{w_{06} = 1}$ $\boxed{w_{36} = -1.5}$ $\boxed{w_{46} = -1.5}$ $\boxed{w_{56} = 2.5}$

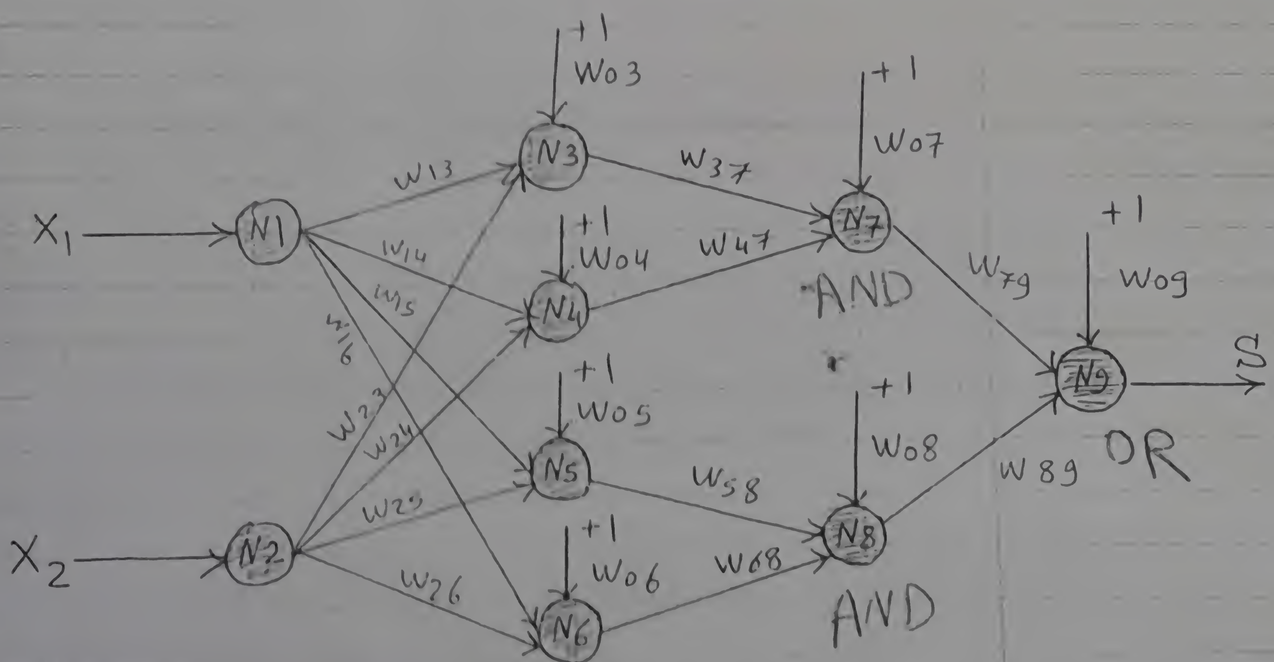
and we get $y_6 = -1.5 f(y_3) - 1.5 f(y_4) + 2.5 f(y_5) + 1$

X_1	X_2	y_3	$f(y_3)$	y_4	$f(y_4)$	y_5	$f(y_5)$	y_6	S	C_1/C_0
0	2	$-1<0$	0	$-1<0$	0	$-1.5<0$	0	$1>0$	1	C_1
0	-2	$3>0$	1	$3>0$	1	$0.5>0$	1	$0.5>0$	1	C_1
2	1	$-2<0$	0	$2>0$	1	$-0.5<0$	0	$-0.5<0$	0	C_0
-3	0	$4>0$	1	$-2<0$	0	$-0.5<0$	0	$-0.5<0$	0	C_0

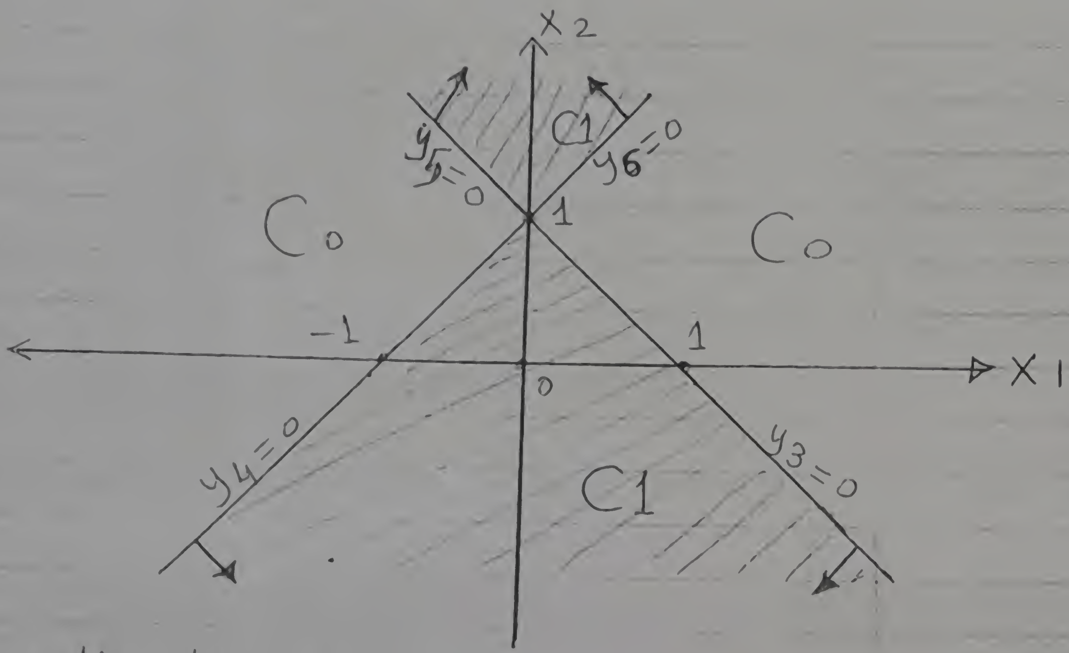
The input pattern (1,2) cannot be properly classified by this network because it lies on the seperation line (2) $y_4 = 0$.

Solution 2

The neural network consists of four layers: an input layer with two neurons N1 and N2; two hidden layers, the first with four neurons N3, N4, N5, and N6, and the second with two neurons N7 and N8, and an output layer with a single neuron N9.
All neurons in the hidden and output layers employ binary threshold activation signals.



* The orientation of the four separation lines are taken as indicated.
* Neurons N7 and N8 perform logic AND operation each and neuron N9 performs a logic OR operation



separation Line $y_3 = 0$

$$-X_1 - X_2 + 1 = 0$$

$$y_3 = w_{13} X_1 + w_{23} X_2 + w_{03}$$
$$= -X_1 - X_2 + 1$$

$\boxed{w_{13} = -1}$ $\boxed{w_{23} = -1}$ $\boxed{w_{03} = 1}$

separation line $y_4 = 0$

$$X_1 - X_2 + 1 = 0$$

$$y_4 = w_{14} X_1 + w_{24} X_2 + w_{04}$$
$$= X_1 - X_2 + 1$$

$\boxed{w_{14} = 1}$ $\boxed{w_{24} = -1}$ $\boxed{w_{04} = 1}$

separation line $y_5 = 0$

$$X_1 + X_2 - 1 = 0$$

$$y_5 = w_{15} X_1 + w_{25} X_2 + w_{05}$$
$$= X_1 + X_2 - 1$$

$\boxed{w_{15} = +1}$ $\boxed{w_{25} = +1}$ $\boxed{w_{05} = -1}$

Separation line $y_6 = 0$

$$-x_1 + x_2 - 1 = 0$$

$$y_6 = w_{16} x_1 + w_{26} x_2 + w_{06}$$

$$= -x_1 + x_2 - 1$$

$\boxed{w_{16} = -1}$ $\boxed{w_{26} = 1}$ $\boxed{w_{06} = -1}$

(N8,N7) performs a logic AND operation on the signals $f(y_3)$ and $f(y_4)$ produced by neurons N3 and N4, respectively.

Activation of N7, $y_7 = w_{37} f(y_3) + w_{47} f(y_4) + w_{07}$

For $f(y_3) = 0$ and $f(y_4) = 0$

$$y_7 = w_{07} < 0$$

for $f(y_3) = 0$ and $f(y_4) = 1$

$$y_7 = w_{47} + w_{07} < 0$$

For $f(y_3) = 1$ and $f(y_4) = 0$

$$y_7 = w_{37} + w_{07} < 0$$

For $f(y_3) = 1$ and $f(y_4) = 1$

$$y_7 = w_{37} + w_{47} + w_{07} > 0$$

$f(y_3)$	$f(y_4)$	$f(y_3) \cdot f(y_4)$
0	0	0
0	1	0
1	0	0
1	1	1

we choose: $\boxed{w_{37} = 1}$ $\boxed{w_{47} = 1}$ $\boxed{w_{07} = -1.5}$

and we get $y_7 = f(y_3) + f(y_4) - 1.5$

Activation of N8, $y_8 = w_{58} f(y_5) + w_{68} f(y_6) + w_{08}$

we choose

$\boxed{w_{58} = 1}$ $\boxed{w_{68} = 1}$ $\boxed{w_{08} = -1.5}$

$f(y_5)$	$f(y_6)$	$f(y_5) \cdot f(y_6)$
0	0	0
0	1	0
1	0	0
1	1	1

and we get

$$y_8 = f(y_5) + f(y_6) - 1.5$$

Neuron N9 performs a logic OR operation on the signals $f(y_7)$ and $f(y_8)$ produced by neurons N7 an N8, respectively.

Activation of N9, $y_9 = w_{79} f(y_7) + w_{89} f(y_8) + w_{09}$

$f(y_7)$	$f(y_8)$	$f(y_7) + f(y_8)$
0	0	0
0	1	1
1	0	1
1	1	1

For $f(y_7) = 0$ and $f(y_8) = 0$,
$y_9 = w_{09} < 0$

for $f(y_7) = 0$ and $f(y_8) = 1$,
$y_9 = w_{89} + w_{09} > 0$

For $f(y_7) = 1$ and $f(y_8) = 0$;
$y_9 = w_{79} + w_{09} > 0$

For $f(y_7) = 1$ and $f(y_7) = 1$;
$y_9 = w_{79} + w_{89} + w_{09} > 0$

We choose $\boxed{w_{79} = 1}$ $\boxed{w_{89} = 1}$ $\boxed{w_{09} = -0.5}$
and we get

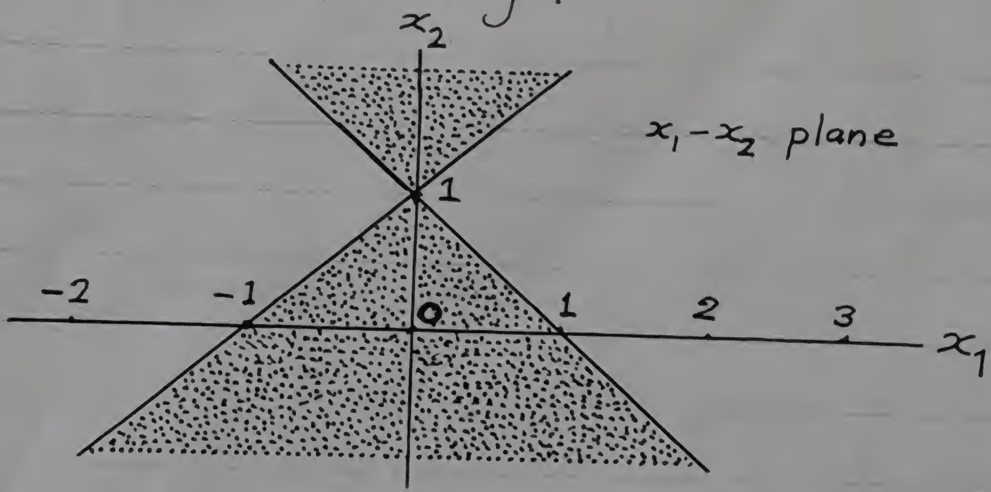
$$y_9 = f(y_7) + f(y_8) - 0.5$$

X_1	X_2	y_3	$f(y_3)$	y_4	$f(y_4)$	y_5	$f(y_5)$	y_6	$f(y_6)$	y_7	$f(y_7)$	y_8	$f(y_8)$	y_9	S	Co/C
0	2	$-1<0$	0	$-1<0$	0	$1>0$	1	$1>0$	1	$-0.5<0$	0	$0.5>0$	1	$0.5>0$	1	C1
0	-2	$3>0$	1	$3>0$	1	$-3<0$	0	$-3<0$	0	$0.5>0$	1	$-1.5<0$	0	$0.5>0$	1	C1
2	1	$-2<0$	0	$2>0$	1	$2>0$	1	$-2<0$	0	$-0.5<0$	0	$-0.5<0$	0	$-0.5<0$	0	Co
-3	0	$4>0$	1	$-2<0$	0	$-4<0$	0	$2>0$	1	$-0.5<0$	0	$-0.5<0$	0	$-0.5<0$	0	Co

The input pattern (1, 2) cannot be properly classified by this network because it lies on the seperation line $y_6 = 0$

Problem 2

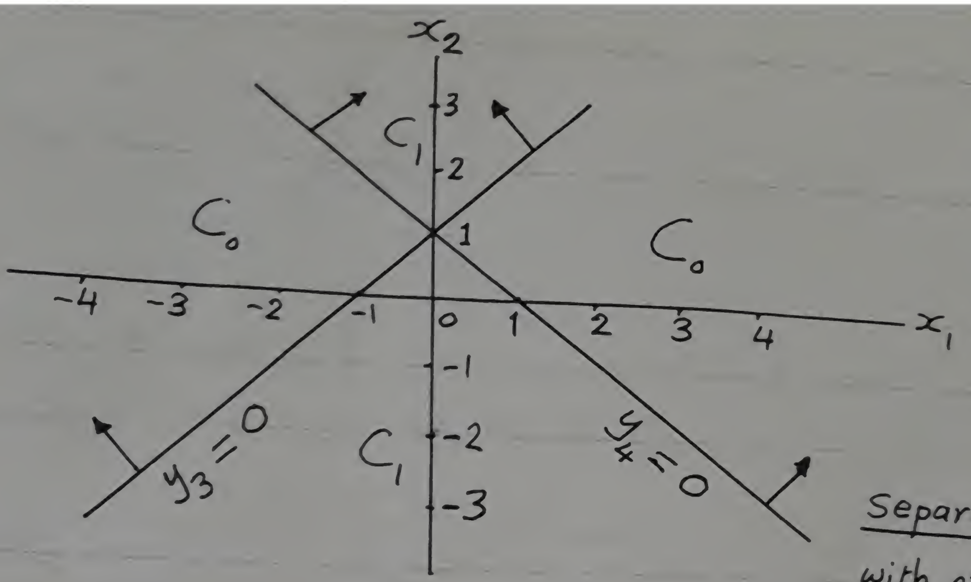
Design a neural network, with two inputs x_1 and x_2 and a single output s, that behaves as a two-class data classifier. On the x_1-x_2 plane, shown below, all input patterns (x_1, x_2) inside the two shaded areas are identified by an output value $s = 1$, whereas all input patterns outside these areas are identified by $s = 0$. How will your network classify the input patterns $(0,2)$, $(0,-2)$, $(2,1)$, and $(-3,0)$? Can the network properly classify the input pattern $(1,2)$? Why?



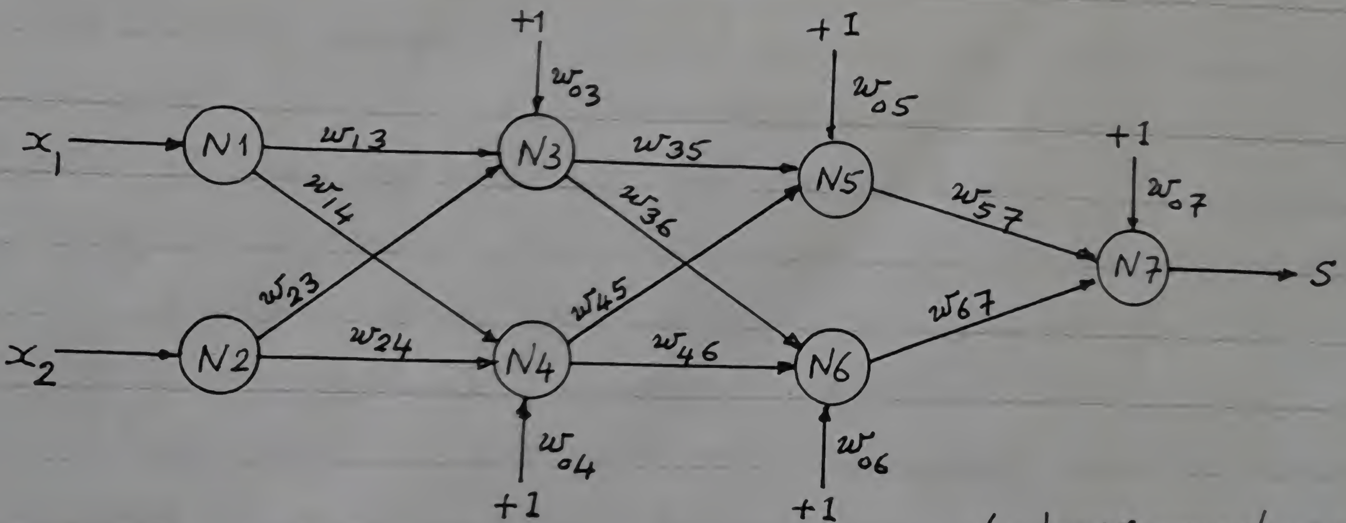
Solution

The neural network consists of four layers: an input layer with two neurons N1 and N2; two hidden layers, the first with two neurons N3 and N4 and the second with two neurons N5 and N6; and an output layer with a single neuron N7.

The orientations of the two separation lines are taken as indicated.



Separation lines with orientations



4-layer neural network

- Neuron N5 performs logic operation $f(y_3)f(y_4)$
- Neuron N6 performs logic operation $f'(y_3)f'(y_4)$
- Neuron N7 performs logic operation $f(y_3)f(y_4) + f'(y_3)f'(y_4)$
  so that $s = f(y_3) \odot f(y_4)$ (XNOR operation)

All neurons in the hidden and output layers employ <u>binary threshold</u> activation signals.

Separation line $y_3 = 0$ (slope $= +1$)

$$x_2 = x_1 + 1 \longrightarrow -x_1 + x_2 - 1 = 0$$

$$y_3 = w_{13} x_1 + w_{23} x_2 + w_{03}$$
$$= -x_1 + x_2 - 1$$

$$\boxed{w_{13} = -1 \qquad w_{23} = 1 \qquad w_{03} = -1}$$

Separation line $y_4 = 0$ (slope $= -1$)

$$x_2 = -x_1 + 1 \longrightarrow x_1 + x_2 - 1 = 0$$
$$y_4 = w_{14} x_1 + w_{24} x_2 + w_{04}$$
$$= x_1 + x_2 - 1$$

$$\boxed{w_{14} = 1 \qquad w_{24} = 1 \qquad w_{04} = -1}$$

The $x_1 - x_2$ plane is divided by the two separation lines into two regions C_1 (the interior of the shaded areas) and C_0 (the exterior of the shaded areas). All points lying in C_1 make the activation of the output neuron y_7 positive and the output $s = 1$, whereas all points lying in C_0 make y_7 negative and $s = 0$.

The part of the network consisting of the second hidden layer (N5 and N6) and the output layer (N 7) performs a logic XNOR operation on the signals $f(y_3)$ and $f(y_4)$ produced by neurons N3 and N4, respectively; that is

$$s = f(y_3) \odot f(y_4)$$

To satisfy this operation, neuron N5 performs a logic AND operation on $f(y_3)$ and $f(y_4)$, neuron N6 performs a logic AND operation on $f'(y_3)$ and $f'(y_4)$, and neuron N7 performs a logic OR operation $f(y_3)f(y_4) + f'(y_3)f'(y_4)$; remember that

$$f(y_3) \odot f(y_4) = f(y_3)\, f(y_4) + f'(y_3)\, f'(y_4)$$

Activation of neuron N5 :

$$y_5 = w_{35} f(y_3) + w_{45} f(y_4) + w_{05}$$

For $f(y_3) = 0$ and $f(y_4) = 0$,

$$y_5 = w_{05} < 0$$

For $f(y_3) = 0$ and $f(y_4) = 1$,

$$y_5 = w_{45} + w_{05} < 0$$

For $f(y_3) = 1$ and $f(y_4) = 0$,

$$y_5 = w_{35} + w_{05} < 0$$

For $f(y_3) = 1$ and $f(y_4) = 1$,

$$y_5 = w_{35} + w_{45} + w_{05} > 0$$

$f(y_3)$	$f(y_4)$	$f(y_3)\, f(y_4)$
0	0	0
0	1	0
1	0	0
1	1	1

We choose

$$\boxed{w_{35} = 1 \qquad w_{45} = 1 \qquad w_{05} = -1.5}$$

and we get

$$y_5 = f(y_3) + f(y_4) - 1.5$$

Activation of neuron N6 :

$$y_6 = w_{36} f(y_3) + w_{46} f(y_4) + w_{06}$$

For $f(y_3) = 0$ and $f(y_4) = 0$,

$$y_6 = w_{06} > 0$$

For $f(y_3) = 0$ and $f(y_4) = 1$,

$$y_6 = w_{46} + w_{06} < 0$$

For $f(y_3) = 1$ and $f(y_4) = 0$,

$$y_6 = w_{36} + w_{06} < 0$$

For $f(y_3) = 1$ and $f(y_4) = 1$,

$$y_6 = w_{36} + w_{46} + w_{06} < 0$$

$f(y_3)$	$f(y_4)$	$f'(y_3)\, f'(y_4)$
0	0	1
0	1	0
1	0	0
1	1	0

We choose

$$\boxed{w_{36} = -1 \qquad w_{46} = -1 \qquad w_{06} = 0.5}$$

and we get

$$y_6 = -f(y_3) - f(y_4) + 0.5$$

Activation of neuron N7:

$$y_7 = w_{57} f(y_5) + w_{67} f(y_6) + w_{07}$$

For $f(y_5) = 0$ and $f(y_6) = 0$,

$$y_7 = w_{07} < 0$$

For $f(y_5) = 0$ and $f(y_6) = 1$,

$$y_7 = w_{67} + w_{07} > 0$$

For $f(y_5) = 1$ and $f(y_6) = 0$,

$$y_7 = w_{57} + w_{07} > 0$$

For $f(y_5) = 1$ and $f(y_6) = 1$,

$$y_7 = w_{57} + w_{67} + w_{07} > 0$$

$f(y_5)$	$f(y_6)$	$f(y_5) + f(y_6)$
0	0	0
0	1	1
1	0	1
1	1	1

$f(y_5) = f(y_3) f(y_4)$
$f(y_6) = f'(y_3) f'(y_4)$
$f(y_5) + f(y_6) = s$

We choose

$$\boxed{w_{57} = 1 \qquad w_{67} = 1 \qquad w_{07} = -0.5}$$

and we get

$$y_7 = f(y_5) + f(y_6) - 0.5$$

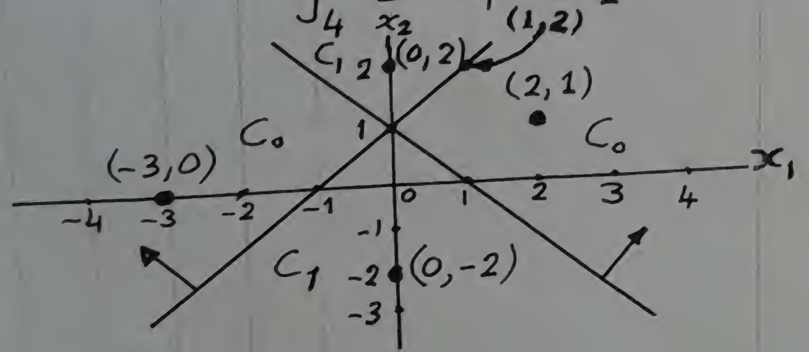
The following table shows how the four input patterns (0,2), (0,-2), (2,1), and (-3,0) are classified; the two input patterns (0,2) and (0,-2) lie in the region C_1 (where $s = 1$) and the other two input patterns (2,1) and (-3,0) lie in the region C_0 (where $s = 0$).

The input pattern (1,2) cannot be properly classified by this network because it lies on the separation line $y_3 = 0$ (or $-x_1 + x_2 - 1 = 0$).

(x_1, x_2)	y_3	$f(y_3)$	y_4	$f(y_4)$	y_5	$f(y_5)$	y_6	$f(y_6)$	y_7	s	C_0/C_1
$(0,2)$	$1>0$	1	$1>0$	1	$0.5>0$	1	$-1.5<0$	0	$0.5>0$	1	C_1
$(0,-2)$	$-3<0$	0	$-3<0$	0	$-1.5<0$	0	$0.5>0$	1	$0.5>0$	1	C_1
$(2,1)$	$-2<0$	0	$2>0$	1	$-0.5<0$	0	$-0.5<0$	0	$-0.5<0$	0	C_0
$(-3,0)$	$2>0$	1	$-4<0$	0	$-0.5<0$	0	$-0.5<0$	0	$-0.5<0$	0	C_0

$$y_3 = -x_1 + x_2 - 1$$

$$y_4 = x_1 + x_2 - 1$$



$$y_5 = f(y_3) + f(y_4) - 1.5$$

$$y_6 = -f(y_3) - f(y_4) + 0.5$$

$$y_7 = f(y_5) + f(y_6) - 0.5$$

Check

$$f(y_5) = f(y_3)f(y_4)$$

$$f(y_6) = f'(y_3)f'(y_4)$$

$$s = f(y_5) + f(y_6)$$

$$= f(y_3) \odot f(y_4)$$